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Spacetime Physics

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The Geometry of Spacetime

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1. Parable of the Surveyors

Once upon a time there was a Daytime surveyor who measured off the king's lands. He took his directions of north and east from a magnetic compass needle. Eastward directions from the center of the town square he measured in meters (x in meters). Northward directions were sacred and were measured in a different unit, in miles (y in miles). His records were complete and accurate and were often consulted by the Daytimers. (See Fig. 1.)

Daytime surveyor uses magnetic north

Nighttimers used the services of another surveyor. His north and east directions were based on the North Star. He too measured distances eastward from the center of the town square in meters (x' in meters) and sacred distances north in miles (y' in miles). His records were complete and accurate. Every corner of a plot appeared in his book with its two coordinates, x' and y'.

Nighttime surveyor uses North Star north

One fall a student of surveying turned up with novel openmindedness. Contrary to all previous tradition he attended both of the rival schools operated by the two leaders of surveying. At the day school he learned from one expert his method of recording the location of the gates of the town and the corners of plots of land. At night school he learned the other method. As the days and nights passed the student puzzled more and more in an attempt to find some harmonious relationship between the rival ways of recording location. He carefully compared the records of the two surveyors on the locations of the town gates relative to the center of the town square:

Table 1. Two different sets of records for the same points.

Place	Daytime surveyor's axes oriented to magnetic north (x in meters; y in miles)		Nighttime surveyor's axes oriented to the North Star (x' in meters; y' in miles)	
Town square	0	0	0	0
Gate A	x_A	y_A	x'_A	y'_A
Gate B	x_B	y_B	x'_B	y'_B
Other gates	...	...	...	...

In defiance of tradition, the student took the daring and heretical step to convert northward measurements, previously expressed always in miles, into meters by multiplication with a constant conversion factor, k. He then discovered that the quantity $[(x_A)^2 + (ky_A)^2]^{1/2}$ based on Daytime measurements of the position of gate A had exactly the same numerical value as the quantity

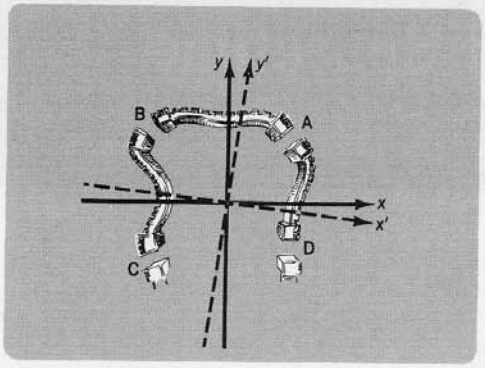


Fig. 1. The town and its gates, showing coordinate axes used by two different surveyors.

$[(x_A')^2 + (ky_A')^2]^{1/2}$ computed from the readings of the Nighttime surveyor for gate A. He tried the same comparison on the readings computed from the recorded positions of gate B, and found agreement here too. The student's excitement grew as he checked his scheme of comparison for all the other town gates and found everywhere agreement. He decided to give his discovery a name. He called the quantity

Discovery: invariance of distance

$$[(x)^2 + (ky)^2]^{1/2} \tag{1}$$

the *distance* of the point (x, y) from the center of town. He said that he had discovered the *principle of the invariance of distance;* that one gets exactly the same distances from the Daytime coordinates as from the Nighttime coordinates, despite the fact that the two sets of surveyors' numbers are quite different.

This story illustrates the naive state of physics before the discovery of *special relativity* by Einstein of Bern, Lorentz of Leiden, and Poincaré of Paris. How naive?

1. Surveyors in this mythical kingdom measured northward distances in a sacred unit, the mile, different from the unit used in measuring eastward distances. Similarly, people studying physics measured time in a sacred unit, the second, different from the unit used in measuring space. No one thought of using the same unit for both, or of what one could learn by squaring and combining space and time coordinates when both were measured in meters. The conversion factor between seconds and meters, namely the speed of light, $c = 2.997925 \times 10^8$ meters per second, was regarded as a sacred number. It was not recognized as a mere conversion factor like the factor of conversion between miles and meters—a factor that arose out of historical accidents alone, with no deeper physical significance.
2. In the parable the northbound coordinates, y and y', as recorded by the two surveyors did not differ very much because the two directions of north were separated only by the small angle of 10 degrees. At first our mythical student thought the small differences between y and y' were due to surveying error alone. Analogously, people have thought of the time between the explosion of two firecrackers as the same, by whomever observed. Only in 1905 did we learn that the time difference between the second event and the first, or "reference event," really has dif-

ferent values, t and t', for observers in different states of motion. Think of one observer standing quietly in the laboratory. The other observer zooms by in a high-speed rocket. The rocket comes in through the front entry, goes down the middle of the long corridor and out the back door. The first firecracker goes off in the corridor ("reference event") then the other ("event A"). Both observers agree that the reference event establishes the zero of time and the origin for distance measurements. The second explosion occurs, for example, 5 seconds later than the first, as measured by laboratory clocks, and 12 meters further down the corridor. Then its time coordinate is $t_A = 5$ seconds and its position coordinate is $x_A = 12$ meters. Other explosions and events also take place down the length of the corridor. The readings of the two observers can be arranged as in Table 2.

One observer uses laboratory frame

Another observer uses rocket frame

Table 2. Space and time coordinates of the same events as seen by two observers in relative motion. For simplicity the y and z coordinates are zero, and the rocket is moving in the x direction.

Event	*Coordinates as measured by observer who is*			
	standing (x *in meters*; t *in seconds*)		*moving by in rocket* (x' *in meters*; t' *in seconds*)	
Reference event	0	0	0	0
Event A	x_A	t_A	x'_A	t'_A
Event B	x_B	t_B	x'_B	t'_B
Other events	...	...	...	...

3. The mythical student's discovery of the concept of distance is matched by the Einstein-Poincaré discovery in 1905 of the idea of *interval.* The interval as calculated from the one observer's measurements

Discovery: invariance of interval

(2) $$\text{interval} = [(ct_A)^2 - (x_A)^2]^{1/2}$$

agrees with the interval as calculated from the other observer's measurements

(3) $$\text{interval} = [(ct_A')^2 - (x_A')^2]^{1/2}$$

even though the *separate coordinates* employed in the two calculations *do not* agree. The two observers will find different space and time coordinates for events A, B, C, . . . relative to the same reference event, but when they calculate the Einstein *intervals* between these events, their results will agree. *The invariance of the interval*—its independence from the choice of the reference frame—forces one to recognize that time cannot be separated from space. Space and time are part of the single entity, *spacetime*. The geometry of spacetime is truly four-dimensional. In one way of speaking, the "direction of the time axis" depends upon the state of motion of the observer, just as the directions of the y axes employed by the surveyors depend upon their different standards of "north."

The rest of this chapter is an elaboration of the analogy between surveying in space and relating events to one another in spacetime. Table 3 is a preview of this elaboration. To recognize the unity of space and time one follows the procedure that makes a landscape take on meaning—he looks at it from several angles. This is the reason for comparing space and time coordinates of an event in two *different* reference frames in relative motion.

Table 3. Preview: Elaboration of the parable of the surveyors.

Parable of the surveyors: geometry of space	*Analogy to physics: geometry of spacetime*
The task of the surveyor is to locate the position of a point (gate A) using one of two coordinate systems that are rotated relative to one another.	The task of the physicist is to locate the position and time of an event (firecracker explosion A) using one of two reference frames which are in motion relative to one another.
The two coordinate systems: oriented to magnetic north and to North-Star north.	The two reference frames: the laboratory frame and the rocket frame.
For convenience all surveyors agree to make position measurements with respect to a common origin (the center of the town square).	For convenience all physicists agree to make position and time measurements with respect to a common reference event (explosion of the reference firecracker).
The analysis of the surveyors' results is simplified if x and y coordinates of a point are both measured in the same units, in meters.	The analysis of the physicists' results is simplified if the x and t coordinates of an event are both measured in the same units, in meters.
The *separate* coordinates x_A and y_A of gate A *do not* have the same values respectively in two coordinate systems that are rotated relative to one another.	The *separate* coordinates x_A and t_A of event A *do not* have the same values respectively in two reference frames that are in uniform motion relative to one another.
Invariance of distance. The *distance* $(x_A^2 + y_A^2)^{1/2}$ between gate A and the town square has the same value when calculated using measurements made with respect to either of two rotated coordinate systems (x_A and y_A both measured in meters).	*Invariance of the interval.* The *interval* $(t_A^2 - x_A^2)^{1/2}$ between event A and the reference event has the same value when calculated using measurements made with respect to either of two reference frames in relative motion (x_A and t_A both measured in meters).
Euclidean transformation. Using *Euclidean* geometry, the surveyor can solve the following problem: *Given* the Nighttime coordinates x_A' and y_A' of gate A and the relative inclination of respective coordinate axes, *find* the Daytime coordinates x_A and y_A of the same gate.	*Lorentz transformation.* Using *Lorentz* geometry, the physicist can solve the following problem: *Given* the rocket coordinates x_A' and t_A' of event A and the relative velocity between rocket and laboratory frames, *find* the laboratory coordinates x_A and t_A of the same event.

Measure time in meters

The parable of the surveyors cautions us to use the same unit to measure both distance and time. So use meters for both. Time can be measured in meters. When a mirror is mounted at each end of a stick one-half meter long, a flash of light may be bounced back and forth between these two mirrors. Such a device is a *clock*. This clock may be said to "tick" each time the light flash arrives back at the first mirror. Between ticks the light flash has traveled a round-trip distance of 1 meter. Therefore the unit of time between ticks of this clock is called *1 meter of light-travel time* or more simply *1 meter of time*. (Show that 1 second is approximately equal to 3×10^8 meters of light-travel time.)

One purpose of the physicist is to sort out simple relations between events. To do this here he might as well choose a particular reference frame with respect to which the laws of physics have a simple form. Now, the force of gravity acts on everything near the earth. Its presence complicates the laws of motion as we know them from common experience. In order to eliminate this and other complications, we will, in the next section, focus attention on a freely falling reference frame near the earth. In this reference frame no gravitational forces will be felt. Such a gravitation-free reference frame will be called an *inertial reference frame*. Special relativity deals with the classical laws of physics expressed with respect to an inertial reference frame.

Simplify: Pick freely falling laboratory

The principles of special relativity are remarkably simple. They are very much simpler than the axioms of Euclid or the principles of operating an automobile. Yet both Euclid and the automobile have been mastered—perhaps with insufficient surprise—by generations of ordinary people. Some of the best minds of the twentieth century struggled with the concepts of relativity, not because nature is obscure, but simply because man finds it difficult to outgrow established ways of looking at nature. For us the battle has already been won. The concepts of relativity can now be expressed simply enough to make it easy to think correctly—thus "making the bad difficult and the good easy."† The problem of understanding relativity is no longer one of *learning* but one of *intuition*—a practiced way of seeing. When seen with this intuition, a remarkable number of otherwise incomprehensible experimental results are revealed to be perfectly natural.‡

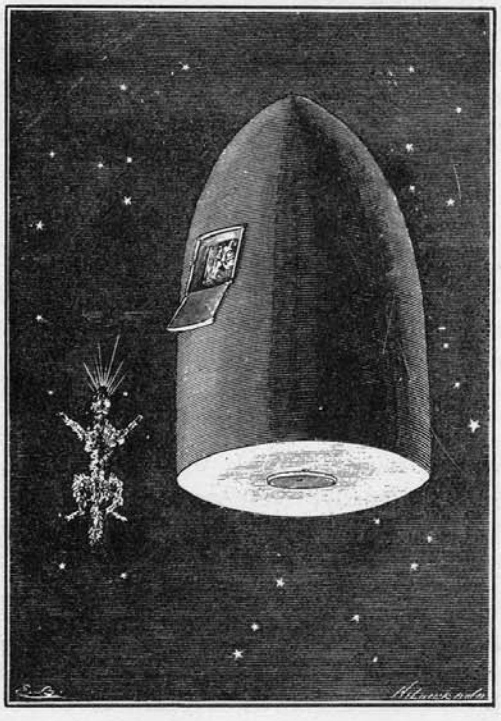
2. The Inertial Reference Frame

Less than a month after the surrender at Appomattox ended the American Civil War (1861–65), the French author Jules Verne began writing *A Trip from the Earth to the Moon* and *A Trip around the Moon*.§ Eminent American cannon designers, so the story goes, cast a great cannon in a pit dug in the earth of Florida with the cannon muzzle pointing skyward. From this cannon is fired a 10-ton projectile containing three men and several animals. As the projectile coasts outward in unpowered flight toward the moon after leaving the cannon, its passengers walk normally inside the projectile on the side

†Einstein, in a similar connection, in a letter to the architect Le Corbusier.

‡For a comprehensive set of references to introductory literature concerning the special theory of relativity, together with several reprints of articles, see *Special Relativity Theory*, Selected Reprints, published for the American Association of Physics Teachers by the American Institute of Physics, 335 East 45th Street, New York 17, New York, 1963.

§Paperback edition published by Dover Publications, New York. Hardcover edition published in the Great Illustrated Classics Series by Dodd, Mead and Company, New York, 1962.



IT WAS THE BODY OF SATELLITE.

Fig. 2. Illustration from an early edition of *A Trip around the Moon.* Satellite is the name of the unfortunate dog.

Passenger felt weight in Jules Verne's space ship

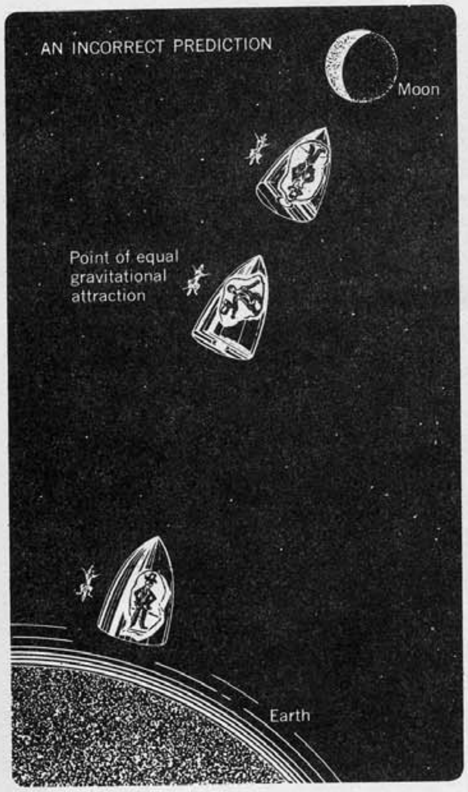
nearer the earth (Fig. 3, A). As the trip continues, the passengers find themselves pressed less and less against the floor of the space ship until finally, at the point where the earth and moon exert equal but opposite gravitational attraction for all objects, the passengers float free of the floor. Later, as the ship nears the moon, they walk around once again, but now against the side of the space ship nearer the moon. Early in the trip one of the dogs in the ship had died from injuries sustained at takeoff. The passengers had disposed of the remains of the dog through a scuttle in the side of the space ship, only to find that the corpse continues to float outside the window during the entire trip.

Paradox of passenger and dog

This story leads to a paradox of crucial importance to relativity. Verne thought it reasonable that the gravitational attraction of the earth would keep a passenger pressed against the earth side of the space ship during the early part of the trip. He also thought it reasonable that the dog should remain next to the ship, since both ship and dog independently follow the same path through space. But if the dog floats *outside* the space ship during the entire trip, why doesn't the passenger float around *inside* the space ship? If the ship were sawed in half would the passenger, now "outside," float free of the floor?

Passenger is weightless in real space ship

Our experience with actual space flights enables us to resolve this paradox. Jules Verne was in error about the motion of the passenger inside the space ship. Like the dog outside the ship, the passenger inside independently follows the same path through space as the space ship itself. Therefore he floats freely relative to the ship during the entire trip (Fig. 3,B). It is true that the gravitational field of the earth acts on the passenger. But it also acts on the space ship. In fact, with respect to the earth, the acceleration of the *spaceship* in the gravitational field of the earth is just *equal* to the acceleration of the



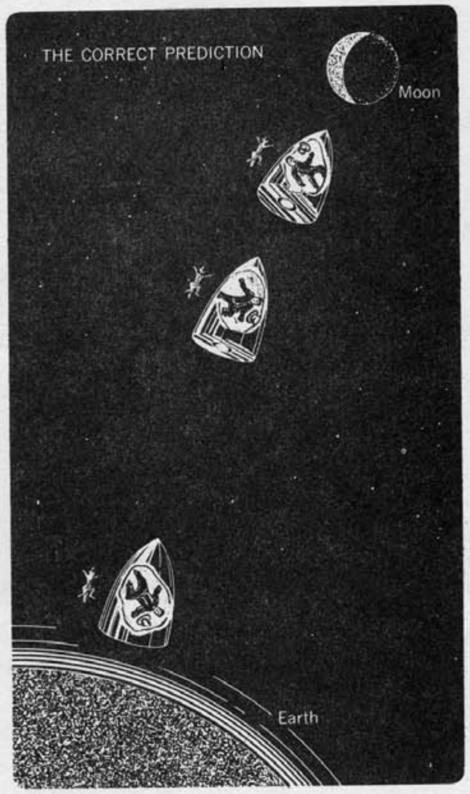


Fig.3, A. Jules Verne believed that a passenger in a free projectile would stand on the side of the projectile nearest to the earth or moon, whichever had greatest gravitational attraction—but that the dog would float along beside the projectile during the entire trip.

Fig 3, B. Correct prediction is that a passenger would float with respect to the projectile during entire trip. Verne was correct about the motion of the dog.

passenger in the gravitational field of the earth. Because of the equality of these accelerations there will be no *relative* acceleration between passenger and space ship. Thus the space ship serves as a reference frame ("inertial reference frame") relative to which the passenger does not experience an acceleration.

To say that the acceleration of the passenger relative to the space ship is zero is not to say that his velocity relative to it is necessarily also zero. He may have jumped from the floor or sprung from the side—in which case he will hurtle across the space and strike the opposite wall. However, when he has zero initial velocity relative to the ship the situation is particularly interesting, for he will also have zero velocity relative to it at all later times. He and the ship will follow identical paths through space. How remarkable that the passenger who cannot see the outside nevertheless moves on this deterministic orbit. Without a way to control his motion and even with his eyes closed he will not touch the wall. How could one do better at eliminating gravitational influences!

A modern space ship carrying a passenger is shot vertically from the earth, rises, and falls back toward the earth (Fig. 4). (The passenger of an elevator car experiences a close approximation to this fall when the elevator cable is cut!) Choose this freely falling space ship as the best possible reference frame in which to do physics. This reference frame is best because, among other

Concept of inertial reference frame

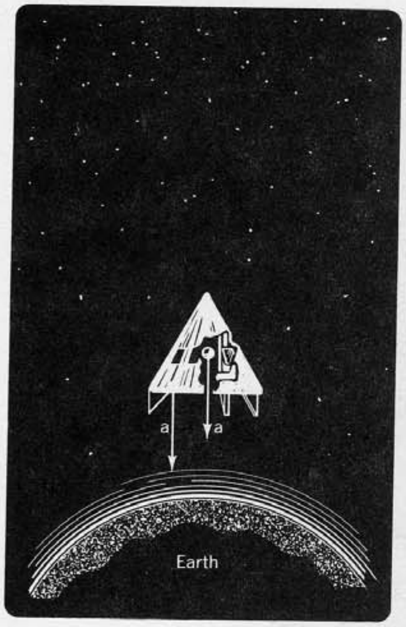


Fig. 4. Space ship in free fall near the earth.

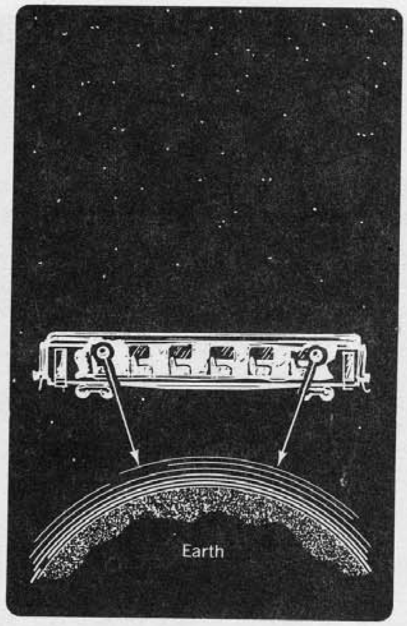


Fig. 5. Railway coach in free fall in *horizontal* position near the earth.

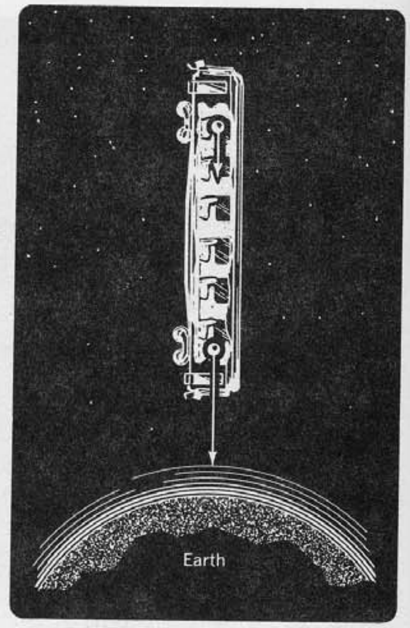


Fig. 6. Railway coach in free fall in *vertical* position near the earth.

things, the laws of motion of a particle are simple in a falling space vehicle. A free particle at rest in the vehicle remains at rest in the vehicle. When the particle is given a gentle push, it moves across the vehicle in a straight line with constant speed. Further experiments show that *all* the laws of mechanics can be expressed simply with respect to a falling space ship. We call such a space ship that rises or falls freely—or more generally moves freely in space—an *inertial reference frame*.

Look at the freely falling space ship from the surface of the earth. There is a simple reason why the free particle at rest relative to the space ship remains at rest in the space ship. This reason is that, with respect to the surface of the earth, the particle and the space ship both fall with the same acceleration (Fig. 4). It is because of this equal acceleration that the *relative* positions of the particle and the space ship do not change if the particle is originally at rest in the space ship.

Earth's pull nonuniform: large space ship is not inertial frame

The definition of an inertial frame requires that *no gravitational forces will be felt in it*. If such a reference frame is to be a space ship near the earth, it cannot be a very large one because widely separated particles within it will be differently affected by the *nonuniform* gravitational field of the earth. For example, particles released side by side will each be attracted toward the center of the earth, so they will move closer together as observed from the falling space ship (Fig. 5). As another example, think of the two particles being released far apart vertically but directly above one another (Fig. 6). Their gravitational accelerations toward the earth will be in the same direction. However the particle nearer the earth will slowly leave the other one behind: the two particles will move farther apart as the space ship falls. In either of these instances the laws of mechanics will not be simple in a very large space ship: the large space ship will not be an inertial frame.

Now, we want the laws of mechanics to look simple in the space ship. Therefore we want to eliminate all relative accelerations produced by external causes—"eliminate" meaning to reduce these accelerations below the limit of detection so that they will not interfere with the more important accelerations we wish to study, such as those produced when two particles collide. This can be done by choosing a space ship that is sufficiently small. The smaller the space ship, the smaller will be the relative accelerations of objects at different points in the space ship. Let someone have instruments for the detection of relative accelerations with any given degree of sensitivity. No matter how fine that sensitivity, the space ship can always be made so small that these perturbing relative accelerations are too small to be detectable. Within these limits of sensitivity the space ship is then an *inertial reference frame*.

Example of space ship small enough to be inertial frame

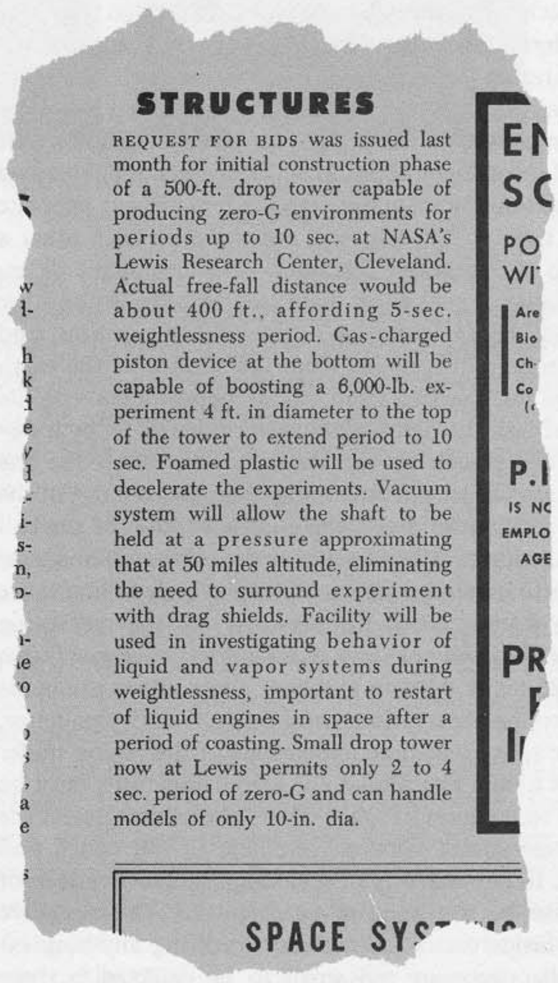
When is a space ship or any other vehicle small enough to be called an inertial reference frame? Or when is the relative acceleration of free particles at opposite ends of the vehicle too slight to be detected? Analyzing the conditions inside one vehicle will serve to illustrate these considerations. A railway coach 25 meters long is dropped in a *horizontal* position from a height of 250 meters onto the surface of the earth (Fig. 5). The time from release to impact is about 7 seconds, or about 21×10^8 meters of light-travel time. Let tiny ball bearings be released initially from rest—and in mid-air—at opposite ends of the coach. Then, during the time of fall, they will move *toward* each other a distance of 10^{-3} meters—the thickness of 9 pages of this book—because of the difference in *direction* of the earth's gravitational pull upon them (see Ex. 32). As another example, assume that the same railway coach is dropped in a *vertical* position, and that the lower end of the coach is initially 250 meters from the surface of the earth (Fig. 6). Again two tiny ball bearings are released from rest at opposite ends of the coach. In this case, during the time of fall, the ball bearings will move *apart* by a distance of 2×10^{-3} meters because of the greater gravitational acceleration of the one nearer the earth. In either of these examples let the measuring equipment in use in the coach be just short of the sensitivity required to detect the relative motion of the ball bearings. Then, with equipment of this degree of sensitivity, and with the limited time of observation, the railway coach—or, to use an earlier example, the freely falling space ship—serves as an inertial reference frame. When the sensitivity of the measuring equipment is increased, then the space ship will not serve as an inertial reference frame unless changes are made. Either the 25-meter domain in which observations are made must be shortened, or the time given to the observations must be decreased. Or, better, some appropriate combination of the space and time dimensions of the region under observation must be cut down. Or, as a final alternative, the whole apparatus must be shot by a rocket (part c of Ex. 32) up to a region of space where one cannot detect the "differential in the gravitational acceleration" between one side of the coach and another—to use one way of speaking. In another way of speaking, the accelerations of the particles *relative to the coach* must be too small to be perceived. These relative accelerations can be measured from inside the coach without observing anything external. Only when these relative accelerations are too small to be detected is there a reference frame with respect to which the laws of motion are simple—an *inertial reference frame*.

Inertial reference frame defined

A reference frame is said to be inertial in a certain region of space and time when, throughout that region of spacetime, and within some specified accuracy, every test particle that is initially at rest remains at rest, and every test particle that is initially in motion continues that motion without change in speed or in *direction.* An inertial reference frame is also called a *Lorentz reference frame.* In terms of this definition, inertial frames are necessarily always *local* ones, that is, inertial in a limited region of spacetime.

Region of spacetime defined

"Region of spacetime." What is the precise meaning of this term? The long narrow railway coach in the example served as a means to probe spacetime for a limited stretch of time and in one or another single direction in space. It can be oriented north-south, or east-west, or up-down. Whatever the orientation, the relative acceleration of the tiny ball bearings released at the two ends can be measured. For all three directions—and for all intermediate directions—let it be found by calculation that the relative drift of the two test particles is half the minimum detectable amount or less. Then throughout a cube of space 25 meters on an edge and for a lapse of time of 7 seconds, test particles moving every which way depart from straight-line motion by undetectable amounts.



STRUCTURES

REQUEST FOR BIDS was issued last month for initial construction phase of a 500-ft. drop tower capable of producing zero-G environments for periods up to 10 sec. at NASA's Lewis Research Center, Cleveland. Actual free-fall distance would be about 400 ft., affording 5-sec. weightlessness period. Gas-charged piston device at the bottom will be capable of boosting a 6,000-lb. experiment 4 ft. in diameter to the top of the tower to extend period to 10 sec. Foamed plastic will be used to decelerate the experiments. Vacuum system will allow the shaft to be held at a pressure approximating that at 50 miles altitude, eliminating the need to surround experiment with drag shields. Facility will be used in investigating behavior of liquid and vapor systems during weightlessness, important to restart of liquid engines in space after a period of coasting. Small drop tower now at Lewis permits only 2 to 4 sec. period of zero-G and can handle models of only 10-in. dia.

Fig. 7. Modern inertial reference frame. From *Engineering Opportunities*, March 1964.

In other words, the reference frame is inertial in a region of spacetime with dimensions

$$(25 \text{ meters} \times 25 \text{ meters} \times 25 \text{ meters of space}) \times (21 \times 10^8 \text{ meters of time})$$

For a discussion of spacetime regions larger than those of local inertial frames, see Chapter 3.

"Test particle." How small must a particle be to qualify as a test particle? It must have so little mass that, within some specified accuracy, its presence will not affect the motion of other nearby particles. In terms of Newtonian mechanics the gravitational attraction of the test particle for other particles must be negligible within the accuracy specified. As an example, consider a particle of mass 10 kilograms. A second and less massive particle placed one-tenth meter from it and initially at rest will, in less than three minutes, undergo a displacement of 10^{-3} meters. Thus the 10-kilogram object is not—in this sense—a test particle. A test particle *responds* to gravitational forces but it does not itself *produce* any significant gravitational force.

Test particle defined

It would be impossible to define an inertial reference frame if it were not for a remarkable feature of nature. Particles of different size, shape, and material in the same location all fall with the *same acceleration* toward the earth. If this were not so, an observer inside a falling space ship would notice a relative acceleration among different particles even when they are close together; at least some of the particles initially at rest would not remain at rest; that is, the space ship would not be an inertial reference frame according to the definition. How sure are we that particles in the *same* location but of *different* substances all fall toward the earth with the same acceleration? According to legend Galileo dropped balls made of different materials from the Leaning Tower of Pisa in order to verify this assumption.† In 1922 Baron Roland von Eötvös checked to an accuracy of five parts in 10^9 that the earth imparts the same acceleration to wood and to platinum. More recently Robert H. Dicke has pointed out that the sun is more suitable than the earth as source for the gravitational acceleration that one will measure (see Ex. 35). The alternation in direction of the sun's pull every 12 hours lends itself to fantastic amplification by resonance. Cylinders of aluminum and gold experience accelerations due to the sun (0.59×10^{-2} meters per second per second) that are the same to three parts in a hundred thousand million (3 in 10^{11}), according to R. H. Dicke and Peter G. Roll.‡ This is one of the most sensitive checks of a fundamental physical principle in all of physics: the identity of the acceleration produced by gravity in every kind of test particle.

Inertial frame is definable because all substances fall with same acceleration

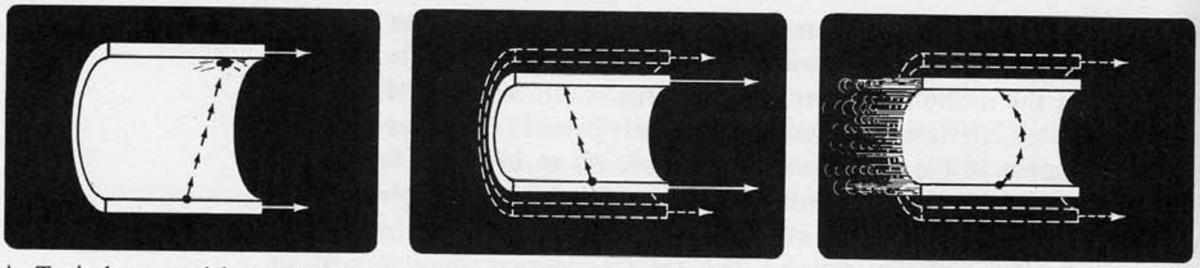
It follows from this principle that a particle made of *any* material can be used as a test particle to determine whether a given reference frame is inertial. A reference frame that is inertial for one kind of test particle will be inertial for all kinds of test particles.

3. The Principle of Relativity

We describe the motion of test particles with reference to a particular reference frame in order to determine whether that frame is inertial. The same test particles and—if they collide—the same collisions may be described with reference to more than one inertial frame. The one reference frame might be carried by a space ship built like a hollow cylinder (Fig. 8,A), the other by a second craft of similar construction just enough smaller to zoom through in-

†On the question whether Galileo actually performed this experiment, see *Physics the Pioneer Science* by Lloyd W. Taylor, (Dover Publications, New York, 1959), Vol. 1, p. 25.

‡See the chapter on experimental relativity by Dicke in *Relativity, Groups, and Topology*, edited by C. and B. DeWitt, (Gordon and Breach, New York, 1964), pp. 173–177 or the book by Dicke *The Theoretical Significance of Experimental Relativity* (Gordon and Breach, New York, 1964).



A. Typical test particle as seen from one inertial reference frame

B. Same test particle as seen from a second inertial reference frame moving with respect to the first one

C. Same test particle as seen from an accelerated—and therefore *non-inertial*—reference frame

Fig. 8. Inertial reference frames versus accelerated reference frame.

side as it catches up with the first and passes it (Fig. 8,B). There is a region of spacetime common to the interior of both vehicles during the time of passing. Flying across this region in one direction or another are numerous test particles. Every track will be straight as plotted with respect to the coordinates of one reference frame—and also with respect to the other frame—because both are *inertial* frames. This straightness in both frames is possible only because *one inertial reference frame has uniform velocity relative to any second and overlapping inertial reference frame.* In contrast, if the second rocket ship is powered so that it accelerates as it passes through the first ship (Fig. 8,C), test particles will follow curved paths—as observed from this second rocket. If the curvature of these paths is detectable with given equipment, then such an accelerated reference frame is a noninertial frame.

Overlapping inertial frames move with uniform relative velocity

In each of two *inertial* frames in uniform relative motion, every test particle that is in motion continues that motion without change in speed or in direction, even though the direction and speed of a given particle will not look the same in both frames. Indeed, we have *defined* the inertial frame in such a way that the following law of mechanics (Newton's first law) is true in every inertial reference frame: "A free particle at rest remains at rest, and a free particle in motion continues that motion without change in speed or in direction." There are additional laws of mechanics. Each of these laws also holds true—according to experiment—in every inertial reference frame.

Do other laws of physics maintain their validity in every inertial frame? In designing electrical circuits for a jet plane must an electrical engineer use different circuit laws because the plane will be moving? Must different electromagnetic laws of radiation be used in designing a radio transmitter for a space probe because of the motion of the probe? If a small proton accelerator together with its targets and particle-detecting equipment is mounted on a railway flatcar, will the interpretation of collision experiments with protons require the use of different laws when the flatcar is moving uniformly than when the flatcar is at rest? As far as we know the answer to these three questions and others like them is "No." In spite of the most diligent search no one has ever found any violation of the following principle:

Relativity: laws of physics are same in every inertial frame

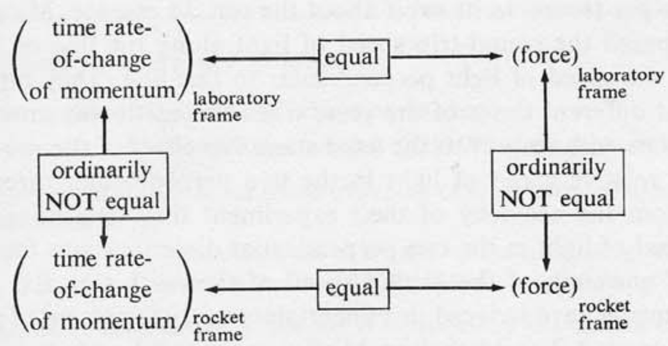
All the laws of physics are the same in every inertial reference frame.

We will call this statement the *principle of relativity*. The principle of relativity says that once the laws of physics have been derived in one inertial reference frame, they can be applied without modification in any other inertial reference frame. Both the *form* of the laws of physics and the *numerical values of the physical constants* that these laws contain are the same in every inertial reference frame. All inertial frames are equivalent in terms of *every* law of physics. Expressed in negative terms, the principle of relativity says that *the laws of physics cannot provide a way to distinguish one inertial frame from another*—any more than the surveyor's tape and level give a means to tell North Star north from magnetic north!

What principle of relativity does NOT say!

Notice what the principle of relativity does *not* say. It does not say that the time between events A and B will appear the same when measured from two different inertial reference frames. Neither does it say that spatial separation between the two events will be the same in the two frames. Ordinarily neither times nor distances will be the same in the two frames—any more than the northward and eastward components of the separation of gates A and B as read by the Daytime surveyor are identical to those recorded by the Nighttime surveyor. In consequence the momentum of a given particle will have a value in one frame that is different from its value in a second frame. Even the time rate-of-change of momentum will ordinarily differ between the two frames. And so will the force. Thus, when studying the motion of a charged particle, two observers in relative motion will not necessarily find the same values for the electric field or the magnetic field acting upon this charged particle. The total force, produced by the electric and magnetic fields together, will differ between the one inertial reference frame and the other.

The physics that is so different between one frame and the other is nevertheless the same in the two frames! Physical quantities differ in *value* between the two frames but fulfill identical *laws*. The time rate-of-change of momentum in one frame is equal to the total force as measured in that frame (Newton's second law). The time rate-of-change of momentum in the second frame is equal to the total force as measured in that second frame:



Not only the laws of mechanics but also the laws of electromagnetism and all other laws of physics hold true as well in one inertial reference frame as in any other inertial reference frame. This is what it means to say that "the laws of physics cannot provide a way to distinguish one inertial reference frame from another."

The laws of electromagnetism hold true as well in one inertial reference frame as in any other inertial reference frame. The numerical value of the speed of light, $c = 2.997925 \times 10^8$ meters per second, is one of the constants which appears in the laws of electromagnetism. According to the principle of relativity this experimental value must be the same in each of two inertial reference frames in uniform relative motion. Has experiment shown this to be true? The answer is yes, but the experiments to date are much less sensitive than they should be for so important a question. For the moment let us pull in our horns and concentrate on a simpler question which *can* be answered definitively. The laws of electromagnetism contain no expressions that depend upon direction. Therefore one expects to find the round-trip speed of a flash of light to be the same whether the flash travels north-south or east-west: the speed of light is *isotropic*. But now let these same flashes of light be observed from a uniformly moving rocket. With respect to this rocket, will the round-trip speed of light not be different for light moving out and back along different lines? The principle of relativity says *no*: the speed of light, which is isotropic in one inertial frame, is also isotropic in all other inertial frames that share the same spacetime region.

Michelson-Morley experiment: speed of light is isotropic in all inertial frames

How strange this result is! We know that the speed of sound in air is the same in all directions if the air is still. But let a stiff wind be blowing—or, to get the same result, move through the still air in an automobile. Then, with respect to the automobile, the "downstream speed of sound" is greater than the "upstream speed of sound." And a simple calculation shows that both these speeds are different from the speed of sound measured across the wind. The round-trip speed of sound measured with respect to the automobile will be different in different directions. The same result is true for every other form of wave motion we know about—except that of light! How can we be so sure that this result is *not* true for experiments with light? Our assurance is based on a series of refined experiments beginning with the classic experiment of A. A. Michelson and E. W. Morley performed after 1880.† They used the earth itself as a moving reference frame. (The earth is effectively an inertial frame for local experiments with light—see Ex. 31.) The earth moves at a speed of about 30 kilometers per second in its orbit about the sun. In essence, Michelson and Morley compared the round-trip speed of light along the line of the earth's motion with the speed of light perpendicular to this line. They repeated this experiment at different times of the year, when the earth was moving in different directions with respect to the fixed stars. No effect of the motion of the earth on the relative speed of light in the two perpendicular directions was observed. From the accuracy of their experiment they determined that the measured speed of light in the two perpendicular directions was the same to a sensitivity of one-sixth of the orbital speed of the earth (see Ex. 33). More recent experiments have reduced this uncertainty to three percent of the orbital speed of the earth.‡ The Michelson-Morley experiment and its modern improvements tell us that in every inertial frame the round-trip speed of light is the same in every direction—the speed of light is *isotropic* in both laboratory and rocket frames as predicted by the principle of relativity. But the principle of relativity says more than this. Not only must the speed of light be isotropic in the laboratory frame—and also isotropic in the rocket frame—but also, if

†A. A. Michelson and E. W. Morley, American Journal of Science, **34,** 333 (1887).

‡T. S. Jaseja, A. Javan, J. Murray, and C. H. Townes, Physical Review, **133,** A1221 (1964).

Table 4. Modern tests to answer the question, "Does the round-trip speed of light differ between one reference frame and another?"

TWO REFERENCE FRAMES

ONE REFERENCE FRAME

The earth moving in one direction around the sun in, say, January.

ANOTHER REFERENCE FRAME

The earth moving in the opposite direction (with respect to the fixed stars) in July.

EXPERIMENTAL RESULTS

RESULT OF THE MICHELSON-MORLEY EXPERIMENTS

Original experiment

Observers in neither frame (may be the *same* observer on earth who repeats the experiment after waiting six months) can detect differences in the round-trip speed of light in any two perpendicular directions greater than one-sixth of the speed of the earth in its orbit.

The more modern experiment

Observers in neither frame can detect differences in the round-trip speed of light in any two perpendicular directions greater than three percent of the speed of the earth in its orbit.

RESULT OF THE KENNEDY-THORNDIKE EXPERIMENT

The round-trip speed of light is the same in one of the seasonal reference frames defined above as in the other reference frame with a sensitivity of about two meters per second.

INTERPRETATION OF THE EXPERIMENTS

THE MORE MODERN MICHELSON-MORLEY EXPERIMENT

The speed of the earth in its orbit about the sun is

30 kilometers per second = 1/10,000 of the speed of light

Thus the *difference* of the round-trip speed of light measured in two perpendicular directions is

less than 3/100 of 1/10,000 of the speed of light
which is less than 3/1,000,000 of the speed of light

Therefore the *principle of relativity* is supported by this modern experiment with a sensitivity of

three parts in a million

THE KENNEDY-THORNDIKE EXPERIMENT

The *difference* of the round-trip speed of light as measured in the two frames is

less than about 2 meters per second
which is less than 1/100,000,000 of the speed of light

Therefore the *principle of relativity* is supported by this experiment with a sensitivity of

one part in a hundred million

Kennedy-Thorndike experiment: speed of light has same numerical value in all inertial frames

this principle is correct, the *numerical value* of this isotropic speed, $c = 2.997925 \times 10^8$ meters per second, must be the same in the rocket frame as in the laboratory frame. Can this prediction also be verified by experiment? This verification was carried out by R. J. Kennedy and E. M. Thorndike about 50 years after Michelson and Morley did their experiment.† Like Michelson and Morley, Kennedy and Thorndike used the earth as a moving reference frame. They tried to detect any variation in the magnitude of the round-trip speed of light as the earth moved in different directions around the sun at different times of the year. From the accuracy of their negative results one can conclude that there is no difference as great as about two meters per second in the round-trip speed of light as between two reference frames with a relative velocity of 60 kilometers per second (twice the speed of the earth in its orbit; see Ex. 34). In the Kennedy-Thorndike experiment the standard of length is the interferometer base itself, a single block of fused quartz kept in a vacuum at a temperature constant to about a thousandth of a degree. The standard of time is provided by the characteristic vibration period associated with a particular green spectral line of a mercury atom. Keeping conditions constant for months constituted the most important single difficulty—and difference—of this Pasadena, California experiment as contrasted to the Cleveland, Ohio Michelson-Morley experiment, where the relevant comparisons (one direction as against another) could be made in the course of a single day. Table 4 summarizes the conclusions of the Michelson-Morley and the Kennedy-Thorndike experiments.

Although the sensitivity of neither of these experiments is as great as that of the Eötvös-Dicke experiment (three parts in a hundred thousand million), the results are nonetheless striking experimental support for the principle of relativity. Happily, there are plans to improve the sensitivity of the Kennedy-Thorndike experiment.‡ This improvement in sensitivity is important. The measurement of time in meters of light-travel time has meaning only if light travels one meter in the same time in all frames. The equality of the speed of light in rocket and laboratory frames provides a simple way (Section 5) to compare clocks between the two frames. This comparison depends for its validity on the null result of the Kennedy-Thorndike experiment.

Structure of spacetime makes LINAC cost \$300,000,000

In 1905 the principle of relativity was a shocking heresy, which offended the intuition and common-sense way of looking at nature of most physicists. It has taken a long time to become accustomed to the apparently absurd idea that one particular speed has the same value measured in two overlapping inertial frames in relative motion. The principle of relativity is used every day in many fields of physics where it is continually under severe tests. For example, the Stanford Linear Electron Accelerator (estimated cost: \$300,000,000) has to be two miles long to push electrons up to a speed that is almost the speed of light (the difference from the speed of light is only 8 parts in 10^{11}). If the pre-Einstein Newtonian laws of mechanics were correct, then the accelerator

†R. J. Kennedy and E. M. Thorndike, Physical Review, **42,** 400 (1932).

‡See Jaseja, Javan, Murray, and Townes, Physical Review, **133,** A1221 (1964). For a careful analysis of the experimental foundations of special relativity see H. P. Robertson, "Postulate versus Observation in the Special Theory of Relativity," Reviews of Modern Physics, **21,** 378 (1949).

would need to be less than one inch long (Ex. 55) to produce electrons with the same speed!

4. The Coordinates of an Event

The inertial reference frame is to a student of physics what the north-south east-west grid of lines in a township is to a surveyor. The surveyor is concerned with position in space. The student of physics is concerned with location of an event in space and in *time*. The Daytime and Nighttime surveyors *could* have dispensed with north-south and east-west coordinates and simply measured the *distance* between any two gates; but at the start they did not even know there was any such quantity as "distance." In the same way in this chapter we could have gone about locating events in spacetime solely by measuring the *intervals* between one event and another, without any regard for "space" and "time" coordinates individually.† However, we have to start as physics did before 1905, without benefit of any concept of interval. This concept will force itself upon our attention as the concept of distance forced itself upon the surveyors. The two men measured north-south and east-west coordinates in two different coordinate systems. Only later did they see the connection ("invariance of the distance") between the very different numbers in their notebooks. Similarly, we will begin with space and time coordinates of events in the laboratory reference frame, and space and time coordinates of the same events in the rocket reference frame. Then there will be a firm basis for concluding that the interval between two events as determined from laboratory numbers is identical with the interval between the same two events as calculated from the very different rocket readings ("invariance of the interval").

Why use coordinates?

The fundamental concept in surveying is a *place*. The fundamental concept in physics is an *event*. An event is specified not only by a place but also by a time of happening. Some examples of events are: emission of particles or flashes of light (explosions), reflection or absorption of particles or light flashes, collisions, and near-collisions called *coincidences*.

Event defined

How can one determine the place and time at which an event occurs in a given inertial reference frame? Think of constructing a frame by assembling meter sticks into a cubical latticework similar to the "jungle gym" seen on playgrounds (Fig. 9). At every intersection of this latticework fix a clock. These clocks can be constructed in any way, but are *calibrated* in meters of light-travel time. In Section 1 we discussed how to obtain such a calibration by bouncing a flash of light back and forth between two mirrors one-half meter apart. This mirror clock is said to "tick" each time the light flash arrives back at the first mirror. Between ticks the light flash travels a round-trip distance of 1 meter: Call the unit of time between ticks *1 meter of light-travel time*, or more simply, *1 meter of time*. The speed of light in conventional units has the measured value $c = 2.997925 \times 10^8$ meters per second. Light will travel 1

Latticework of clocks

†Such a treatment is presented by Robert F. Marzke and John A. Wheeler in *Gravitation and Relativity*, edited by H.-Y. Chiu and W. F. Hoffmann, (W. A. Benjamin, New York, 1964).

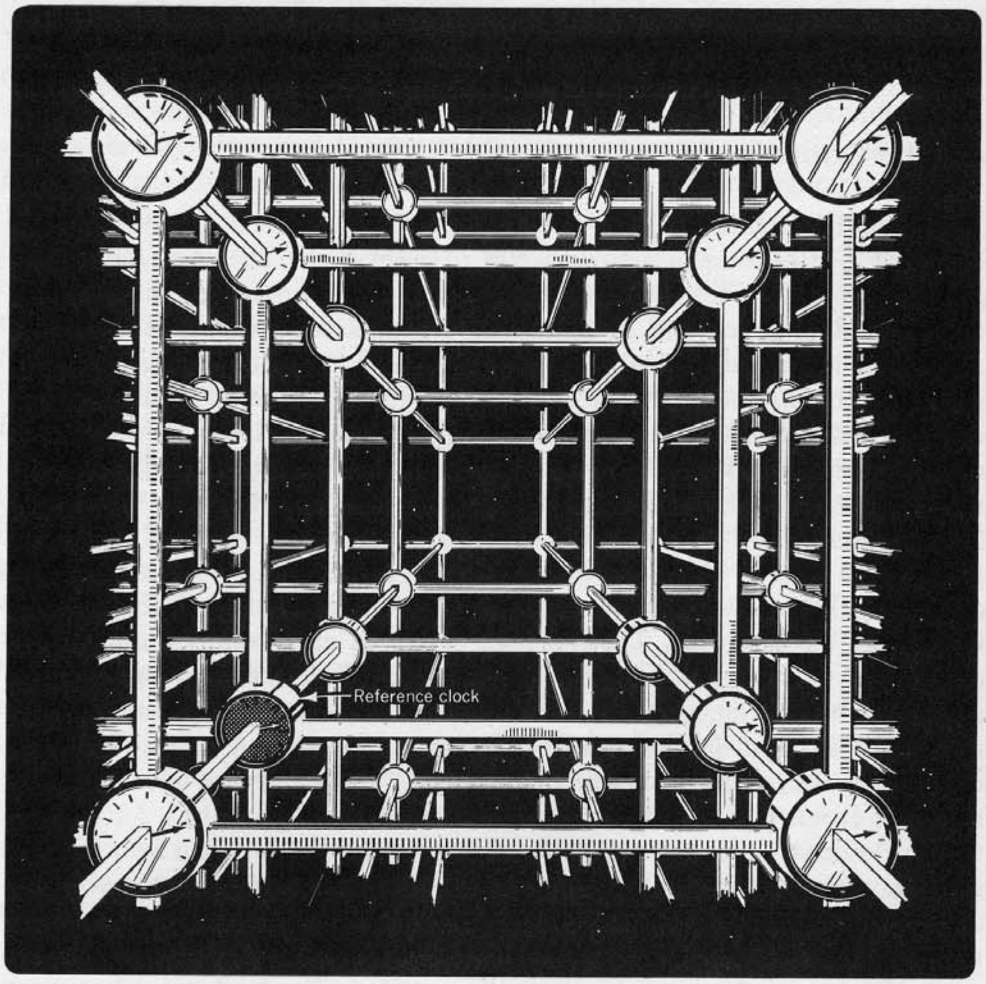


Fig. 9. Latticework of meter sticks and clocks.

meter in the time, 1 meter/c = 3.335640 × 10^{-9} seconds. Hence *1 meter of light-travel time is equal to 3.335640 × 10^{-9} seconds*—about 3.3 *nanoseconds* in the terminology of high-speed electronic circuits! We assume that *every* clock in the latticework, whatever its construction, has been calibrated in meters of light-travel time.

Synchronizing clocks in lattice

How are the different clocks in the lattice to be *synchronized* with one another? As follows: Pick one of the clocks in the lattice as the standard of time and take it to be the *origin* of an x, y, z coordinate system. Start this reference clock with its pointer at $t = 0$. At this instant let it send out a flash of light that spreads in all directions. Call this flash of light the *reference flash.* When the reference flash gets to a clock 5 meters away, we want that clock to read 5 meters of light-travel time. So an assistant *sets* that clock to 5 meters of time long before the experiment begins, *holds* it at 5 meters, and *releases* it only when the reference flash arrives. When assistants at all the clocks in the lattice have followed this procedure (each setting his clock to a time in meters equal to his own distance from the reference clock and starting it when the light flash arrives), the clocks in the lattice are said to be *synchronized.*

There are other possible ways to synchronize clocks. For example, an extra portable clock could be set to the reference clock at the origin and then carried around the lattice in order to set the rest of the clocks. However, this procedure involves a *moving* clock. We will see later that a moving clock runs at a different rate as measured by clocks in the lattice than do clocks that remain at rest in the lattice. The portable clock will not even agree with the first one when it is brought back next to it! (Clock paradox, Ex. 27). However, if we use a moving clock that travels at a speed that is a *very small fraction* of the speed of light, it is only slightly in error, and this second method of synchronization will give very nearly the same result as the first—and standard—method. Moreover the error can be made as small as desired by carrying the portable clock about sufficiently slowly.

Latticework used to measure the four coordinates of event

The latticework of synchronized clocks can be used to determine the location and time at which any given event occurs. The *location* of the event will be taken to be the position of the clock nearest the event. The *time* of the event will be taken to be the time recorded on the lattice clock nearest the event. The *coordinates* of an event will then consist of four numbers: three numbers that specify the *spatial position* of the clock nearest the event and the *time* (in meters) when the event occured as recorded by that clock. The clocks, if they are installed by a foresighted experimenter, will be *recording* clocks. Each will be able to detect the occurrence of an event (passage of a light flash or particle). Each will punch onto a card the nature of the event, the time of the event and the location of the clock. The cards can then be collected from all the clocks and later analyzed, perhaps by automatic equipment.

Lattice spacing depends on scale of physics under study

Why a lattice built of rods that are *one* meter long? When a clock in this lattice punches out a card, one will not know whether the event so recorded is 0.4 meters to the left of the clock, for instance, or 0.2 meters to the right. The location of the event will be uncertain by some substantial fraction of a meter. The time of the event will also be unknown within some appreciable fraction of a meter of light-travel time. This accuracy, however, is quite adequate for observing the passage of a rocket. It is extravagantly good for measurements on planetary orbits—it would even be reasonable to increase the lattice spacing from 1 meter to hundreds of meters. Neither 100 meters nor 1 meter is a lattice spacing suitable for studying the tracks of particles from a high-energy accelerator. There a centimeter or a millimeter would be more appropriate. The location and time of an event can be determined to whatever accuracy is desired by constructing a lattice with sufficiently small spacing.

Observer defined

In relativity we often speak about "the observer." Where is this observer? At one place or all over the place? *The word "observer" is a shorthand way of speaking about the whole collection of recording clocks* associated with one inertial frame of reference. No one real observer could easily do what we ask of the "ideal observer" in our analysis of relativity. So it is best to think of the observer as the man who goes around picking up the punched cards turned out by all the recording clocks in his employ. This is the sophisticated sense in which we will hereafter be using the phrase "the observer finds such and such."

Clock records reveal motion of particle through lattice

The clocks reveal the motion of a particle through the lattice: each clock that the particle passes punches out the time of passage as well as the space coordinate of this event. How can the path of the particle be described in terms of numbers? By recording the coordinates of these events along this path. Differences between the coordinates of successive events reveal the velocity of the particle. The conventional units for speed, v, are meters per second. However, when time is measured in meters of light-travel time, then speed is expressed in meters of distance covered per meter of time. To avoid confusion, speed expressed in meters per meter will be represented by the Greek letter beta, β. A flash of light moves one meter of distance in one meter of light-travel time, or, $\beta_{\text{light}} = 1$. The speed of other particles in meters per meter represents a fraction of the speed of light. In other words $\beta = v/c$. Here and hereafter c represents the speed of light.

Verifying that lattice furnishes inertial frame

From the motion of test particles through the latticework of clocks—or rather from the records of coincidences punched out by the recording clocks—we can determine whether the latticework constitutes an inertial reference frame. *If* the records show (a) that—within some specified accuracy—a test particle moves consecutively past clocks that lie in a straight line, (b) that the speed β of the test particle calculated from the same records is constant—again, within some specified accuracy—and, (c) that the same results are true for as many test-particle paths as the most industrious observer cares to trace throughout the given region of space and time, *then* the lattice constitutes an inertial reference frame throughout that region of spacetime.

Laboratory and rocket frames; x axes coincide

Once again we have described the motion of test particles with respect to a particular reference frame in order to determine whether that frame is inertial. The same test particles and—if they collide—the same collisions may be described with reference to one inertial frame as well as another. Let two reference frames be two different latticeworks of meter sticks and clocks, one moving uniformly relative to the other, and in such a way that their x axes coincide. Call one of these frames the *laboratory frame* and the other—moving in the positive x direction relative to the laboratory frame—the *rocket frame* (Figs. 10 and 11). The rocket is *unpowered* and coasts along with constant velocity relative to the laboratory. Let the rocket and laboratory latticeworks be *overlapping* in the sense that there is a region of spacetime common to both frames (as described in Section 3 and Fig. 8). Test particles move through this common region of spacetime. From the motion of these test particles as recorded by his own clocks, an observer in each frame verifies that his frame is *inertial*.

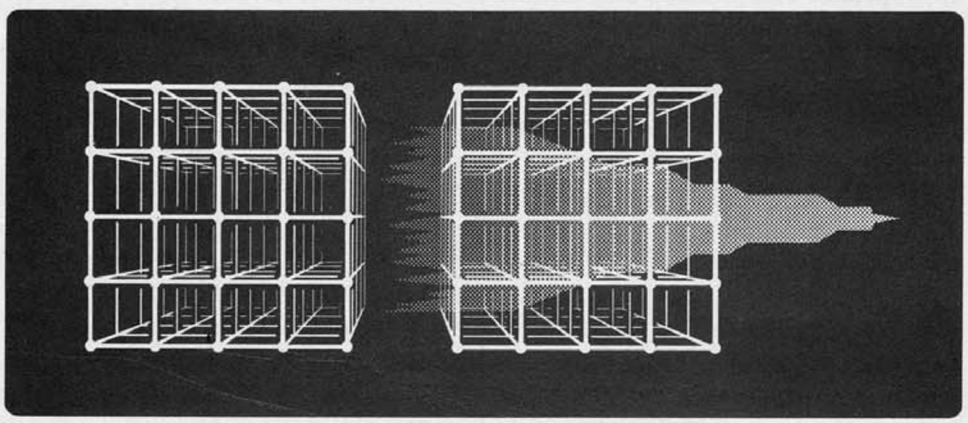


Fig. 10. Laboratory and rocket frames. The two latticeworks intermeshed a second ago.