

# Precise and Full-range Determination of Two-dimensional Equal Loudness Contours

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**Abstract:**

The equal-loudness contours describe the frequency characteristic of sensitivity of our auditory system. A set of the equal-loudness contours based on British data measured in mid-1950 was standardized as ISO 226. However, several research results reported in mid-1980 showed that the set of equal-loudness contours in ISO 226 contained a large error. We, therefore, carried out this project to determine new precise and comprehensive equal-loudness contours and to fully revise the ISO 226. New equal-loudness contours were based on the data of 12 studies, mostly from measurements by the members of this team, reported since 1983. These contours were calculated using a model based on knowledge of our auditory system. Based on the research results, draft standards of the new equal-loudness contours were made and proposed to ISO/TC 43 (Acoustics). The new standard finally was established on August 2003.

**1. Introduction**

An equal-loudness contour is a curve that ties up sound pressure levels having equal loudness as a function of frequency. In other words, it expresses a frequency characteristic of loudness sensation. One of the most famous sets of equal-loudness contours is that reported by Robinson and Dadson [1], which has been standardized as an international standard, ISO226; they have been widely accepted.

In recent years there has been renewed interest in the equal-loudness contours. This interest was triggered by a report by Fastl and Zwicker [2] who noted marked departures from the contours specified by Robinson and Dadson [1] in the region near 400 Hz. Subsequently, the deviations were confirmed by some members of our research team. Figure 1 illustrates the extent of this discrepancy. Here the 40-phon contour measured by Robinson and Dadson [1] is compared with data obtained from

the recent studies reported since 1983. Clearly, in the low frequency region all the newer data deviate systematically from the equal loudness contour by Robinson and Dadson [1]. These differences are nearly as high as 15 dB. Such marked deviations are not only of theoretical importance, they also have practical implications. For example, the current A-weighting for the sound level meter is based on the equal-loudness contour at 40 phons reported by Fletcher and Munson [3] in 1933.

We, therefore, start this international joint research to determine new precise and comprehensive equal-loudness contours over the whole two-dimensional (frequency and sound pressure level) range of audibility and to fully revise ISO 226. To accomplish the purpose, we have carried out the following research items as an international joint research.

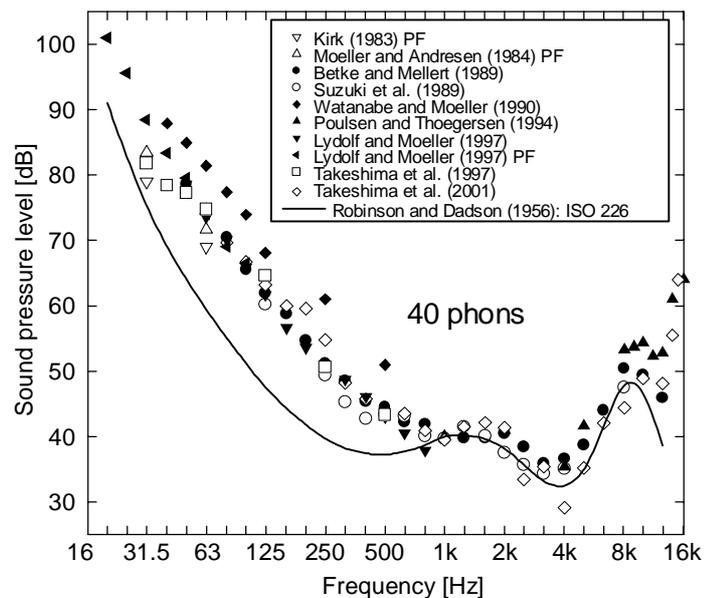
1. Development of unbiased method for measurement of equal-loudness contours
2. Estimation of new two-dimensional equal-loudness contours from all available data
3. Making a proposal of new ISO226 to ISO/TC 43(Acoustics)
4. Considering possibilities for function for better noise evaluation

Based on results of the researches, draft standards of the new equal-loudness contours were made and proposed to ISO/TC 43 (Acoustics). The new standard finally was established on August 2003.

In this report, the result of the research item 2 is mainly described. First, all published studies of equal-loudness contours are reviewed to select basic data for new equal-loudness contours. Then new two-dimensional equal-loudness contours are estimated using a model based on knowledge of our auditory system developed by this research team. Finally, features of the new equal-loudness contours are described by the comparison with the classical contours.

## 2. Selection of the Basic Data for New Equal-loudness Contours

Table I lists the 19 studies [1, 3-20] on equal-loudness contours in chronological order. Although our aim is to establish new equal-loudness-level contours under free-field listening conditions, in several studies equal-loudness levels at low frequencies were measured in a pressure field. These studies are also included in Table I because at frequencies lower than a few hundred Hertz equal-loudness levels of pure tones measured in a free field are consistent with those measured in a pressure field [16].



**Fig.1:** Equal-loudness contour of 40 phons  
Solid line shows the contour proposed by Robinson and Dadson[1]. Symbols show experimental data reported since 1983.

Of the 19 studies listed in Table I, we decided that the following three studies be excluded as candidates for the basic data. Kingsury [4] measured equal-loudness levels under monaural listening conditions with a telephone receiver. However, the levels were not calibrated relative to the levels in a free field. Although Whittle *et al.* [7] made their measurements in a pressure field, equal-loudness levels at 3.15, 6.3, 12.5, and 25 Hz were obtained with reference tones set at 6.3, 12.5, 25 and 50 Hz. No comparison was made with a 1-kHz reference tone. As a result of this shortcoming, the equal-loudness levels they measured cannot be expressed directly in phon. Finally, in the study by Müller and Fichtl [14] the loudness of pure tones was based on the category partitioning procedure. Unfortunately, category-scaling procedures are easily influenced by context effects such as stimulus spacing, the frequency of stimulus presentation, as well as the stimulus range and stimulus distribution.

Four studies, Fletcher and Munson[3], Churcher and King[5], Zwicker and Feldtkeller[6], and Robinson and Dadson [1] proposed a complete set of equal-loudness-level contours whereas the remaining studies reported only measured equal-loudness levels. Owing to their importance, these four sets of contours are referred to as *classic* equal-loudness-level contours, whereas the studies published since 1983 are referred to as *recent* experimental data. In spite of some differences among the results of the various studies, it is clear that all of the *recent* data sets exhibit similar trends. By comparison, none of the four sets of *classic* contours coincide acceptably over the whole range of frequencies and levels with the recent data. Therefore, we decided to use the *recent* data as basic data for new equal-loudness contours.

It is natural to draw threshold of hearing as a lower limit of audibility on a figure of equal-loudness contours, and they will be useful to estimate new equal-loudness contours described in the following sections. In the most researches of equal-loudness contours, thresholds of hearing were measured at the same time. The thresholds of hearing generally well coincide among *recent* studies fit well with the threshold curve by Robinson and Dadson [1]. Therefore, we decided to use the threshold data measured by Robinson and Dadson [1]. Moreover, threshold of hearing for pure tones in a free field have been measured in some studies other than those listed in Table I. They are the following studies: Teranishi [21], Brinkmann [22], Vorländer [23], Poulsen and Han [24]. Data from these studies as well as those from the studies in Table I after 1983 were also used to estimate new equal-loudness contours.

### **3. Derivation of New Equal-loudness Contours**

#### **3.1 Model function for drawing equal-loudness contours**

To draw equal-loudness contours from the experimental data, interpolation along the frequency axis is necessary, because all experimental data are discretely given for specific frequencies and for loudness levels. Moreover, as such data also shows some variances among subjects and studies, appropriate smoothing is required. When conducting such interpolation and smoothing, direct fitting of polynomial regression or spline functions to experimental data would, of course, be possible. In such simple method, however, a contour for a loudness level is drawn independently of other contours, resulting in an ill-shaped representation as a set of equal-loudness contours. To avoid this, use of

**Table I:** Chorography of studies on equal-loudness contours

Year	Researchers	Listening condition	Number of subjects(age)	Method	Ref. tone freq. (level)	Test tone freq.
1927	Kingsbury[4]	Earphone	22 (unspecified)	MA	700 Hz (fix)	60-4000 Hz
1933	Fletcher-Munson[3]	Earphone with correction	11 FF (unspecified)	CS	1 kHz (variable)	62-16000 Hz
1937	Churcher-King[5]	FF	10 (unspecified)	CS	1 kHz (fix)	54-9000Hz
1955	Zwicker-Feldtkeller[6]	Earphone with equalizer	8 FF (unspecified)	Modified Békésy	1 kHz (fix)	50-16000Hz
1956	Robinson-Dadson[1]	FF	90(16-63)/30 (ave. 30)	CS	1 kHz (variable)	25-15000Hz
1972	Whittle <i>et al.</i> [7]	PF	20 (ave. 20)	CS	higher freq (fix)	3.15-50 Hz
1983	Kirk[8]	PF	14 (18-25)	RMLSP	63 Hz (fix)	2-63 Hz
1984	Møller-Andresen[9]	PF	20 (18-25)	RMLSP	63 Hz (fix)	2-63 Hz
1989	Betke-Mellert[10]	FF	13-49 (17-25)	CS	1 kHz (fix)	50-12500Hz
1989	Suzuki <i>et al.</i> [11]	FF	9-32 (19-25)	CS	1 kHz (fix)	31.5-16000Hz
1990	Fastl <i>et al.</i> [12]	FF	12 (21-25)	CS	1 kHz (fix)	100-1000Hz
1990	Watanabe-Møller[13]	FF	10-12 (18-30)	Bracketing	1 kHz (fix)	25-1000Hz
1994	Müller-Fichtl[14]	Open headphones	8(21-25)	CP	-----	62.5-10000Hz
1994	Poulsen-Thøgersen[15]	FF	29 (18-25)	Bracketing	1 kHz (fix)	1000-16000Hz
1997	Lydolf-Møller[16]	FF	27 (19-25)	RMLSP	1 kHz (fix)	50-1000Hz
		PF	27 (19-25)	RMLSP	100 Hz (fix)	20-100 Hz
1997	Takeshima <i>et al.</i> [17]	FF	9-30 (19-25)	CS	1 kHz (fix)	31.5-12500Hz
1999	Bellmann <i>et al.</i> [18]	FF	12 (unspecified)	Adaptive 1up-1down	1 kHz (fix)	100-1000Hz
		PF	12 (unspecified)	Adaptive 1up-1down	100 Hz (fix)	16-160 Hz
2001	Takeshima <i>et al.</i> [19]	FF	7-32 (18-25)	RMLSP	1 kHz (fix)	50-16000Hz

knowledge of a loudness function, i.e., a function representing the growth of loudness as a function of sound intensity level or sound pressure level, may be helpful.

To date, several model functions have been proposed to describe the loudness growth as a function of sound pressure. All of these functions are essentially based on the power law and show distinctive differences only at low sound pressure levels. Which function provides a correct description of loudness at low levels is an important unresolved question, because the recent data show that the level difference between the threshold of hearing and 20 phons are generally larger than that between 20 and 40 phons in low frequency region.

We examined which loudness function is the most appropriate to describe the equal-loudness relation between two pure tones with different frequencies [25]. First, we measured equal-loudness levels of 125-Hz pure tone from 70 phons down to 5 phons. Then, these experimental data were fitted to

five loudness functions. Three of five functions could explain the equal-loudness relation well down to 5 phons. We concluded that the following loudness function is the most appropriate for the present purpose because the number of parameters is less than the other two.

$$S = a(p^{2\alpha} - p_t^{2\alpha}) \quad (1)$$

Where  $p$  is the sound pressure of a pure tone,  $p_t$  is threshold of hearing,  $a$  is a dimensional constant,  $\alpha$  is the exponent, and  $S$  is the perceived loudness. This loudness function was proposed by Zwislocki and Hellman [26] and Lochner and Burger [27].

Atteneave [28] argued that there are two different processes used in absolute magnitude estimation (AME) for assessing the functional relation between assigned numbers and the corresponding perceived magnitudes (i.e., loudness) of a tone presented at a certain sound pressure level. One process was denoted a “loudness perception process” and the other was denoted a “number assignment process.” In addition, Atteneave [28] proposed a two-stage model in which the outputs of both processes are described by separate power transformations. Moreover, in an actual auditory system, the sound emitted from a sound source is transformed by a linear transfer function such as a head-related transfer function (HRTF), and the transfer functions of the middle ear including the transfer function at the entrance to the inner ear (Moore *et al.*, 1997). Then, a loudness-rating model through three blocks may be introduced. Therefore, Eq. (1) could be changed as

$$S = b[a\{(Up)^{2\alpha} - (Up_t)^{2\alpha}\}]^\beta, \quad (2)$$

where  $U$  is a transfer function normalized at 1 kHz.,  $a$  and  $\alpha$  is constant and exponent for “loudness perception process”,  $b$  and  $\beta$  are constant and exponent for “number assignment process.” Here, we assume that  $a$  and  $\alpha$  are dependent on frequency since this makes the residual sum of square in the fitting process much less than with constant  $a$  and  $\alpha$ , and the transformation in the number assigning process is independent of frequencies.

Using these assumptions, when the loudness of an  $f$ -Hz comparison tone is equal in perceived magnitude to the loudness of a reference tone at 1 kHz with a sound pressure of  $p_r$ , then the sound pressure of  $p_f$  at the frequency of  $f$  Hz is given by the following function:

$$p_f^2 = \frac{1}{U_f^2} \left\{ (p_r^{2\alpha_r} - p_{rt}^{2\alpha_r}) + (U_f p_{ft})^{2\alpha_f} \right\}^{1/\alpha_f}, \quad (3)$$

where suffixes  $r$  and  $f$  indicate that the parameters denote the sound pressure for the 1-kHz reference tone and the  $f$ -Hz comparison tone, respectively. Obviously,  $U_f$  is unity at the reference frequency (1 kHz). If these frequency-dependent parameters,  $a_f$  and  $U_f$ , are given, a equal-loudness contour for a specific  $p_r$  can be drawn by connecting  $p_f$  as a function of frequency. Therefore, Eq. (3) is a model function representing equal-loudness contours.

### 3.2 Estimation of new equal-loudness contours

A set of equal-loudness contours was estimated by applying Eq. (3) to the data obtained from the 12 *recent* studies. The estimation of the contours was carried out for the frequency range from 20 Hz

to 12.5 kHz. Above 12.5 kHz, equal-loudness-level data are relatively scarce and tend to be very variable.

To fit Eq. (3) to the data, the exponent at 1 kHz ( $\alpha_r$ ) is required. The typical exponent value obtained by the AME method is 0.27. Loudness measures determined by AME are suitable for the output of the two-stage model. On the basis of these and other earlier investigations an exponent of 0.27 was adopted as the value which corresponds to  $\alpha_r\beta$ . According to Zwislocki's [29] measurements,  $\beta$  has a value of 1.08. Thus, if  $\alpha_r\beta = 0.27$ , and  $\beta$  is 1.08, then  $\alpha_r$ , the exponent at 1 kHz, becomes 0.25. At 1 kHz the exponent value of 0.25 is used for the remainder of our computations.

After the exponent at 1 kHz was fixed, then the procedure outlined below was used to estimate the equal-loudness-level contours.

1. To obtain the best-fitting threshold function, at each frequency from 20 Hz to 18 kHz the experimental threshold data were compiled and averaged. Then, the averages were smoothed across frequency by a cubic B-spline function for the frequency range from 20 Hz to 18 kHz. No weighting was used for this procedure. The numerical values calculated for  $p_{fi}$  and  $p_{ri}$  were used in Eq. (3) to obtain the equal-loudness value for a given comparison-reference frequency pair.
2. Equation (3) was then fitted to the experimental loudness-level data at each frequency by the non-linear least-squares method. A computer program package for general-purpose least squares fittings was used for estimating the values of  $\alpha_f$  and  $U_f$ . The estimated  $\alpha_f$  values were smoothed by the cubic B-spline function assuming that  $\alpha_f$  does not change abruptly as a function of frequency.
3. The third step in our process was to re-estimate the values of  $U_f$  at each frequency. Using the values of  $\alpha_f$  obtained at the previous step, the re-estimated values of  $U_f$  were obtained. The re-estimated  $U_f$  values were smoothed by the cubic B-spline function.

A family of equal-loudness contours obtained in this manner is shown in Fig. 2. The contours in the figure show several aspects to be noted. First, owing to the lack of experimental data at high loudness levels, the 90-phon contour does not extend beyond 4 kHz and the 100-phon contour does not extend beyond 1 kHz. Second, because data from only one institute are available, the 100-phon contour is drawn by a dotted line. Third, owing to the lack of experimental data between 20 phons and the

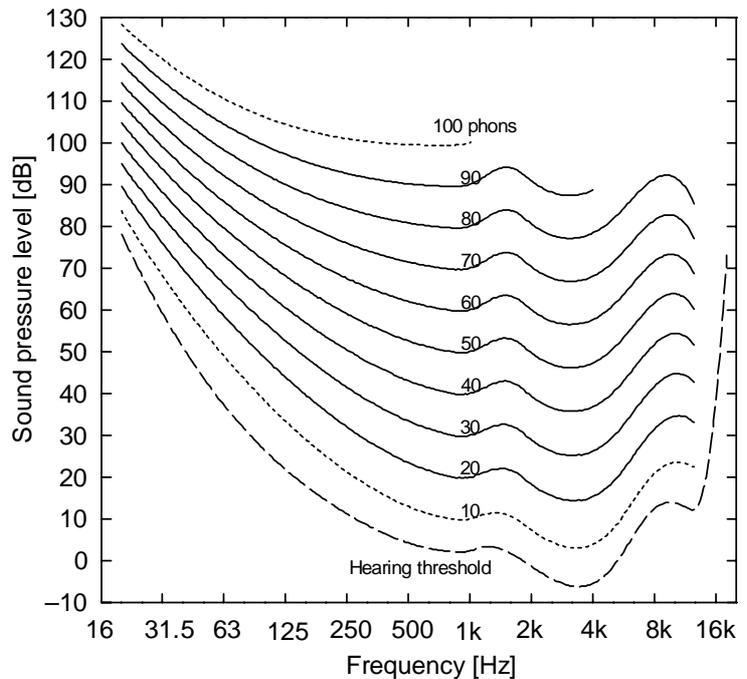
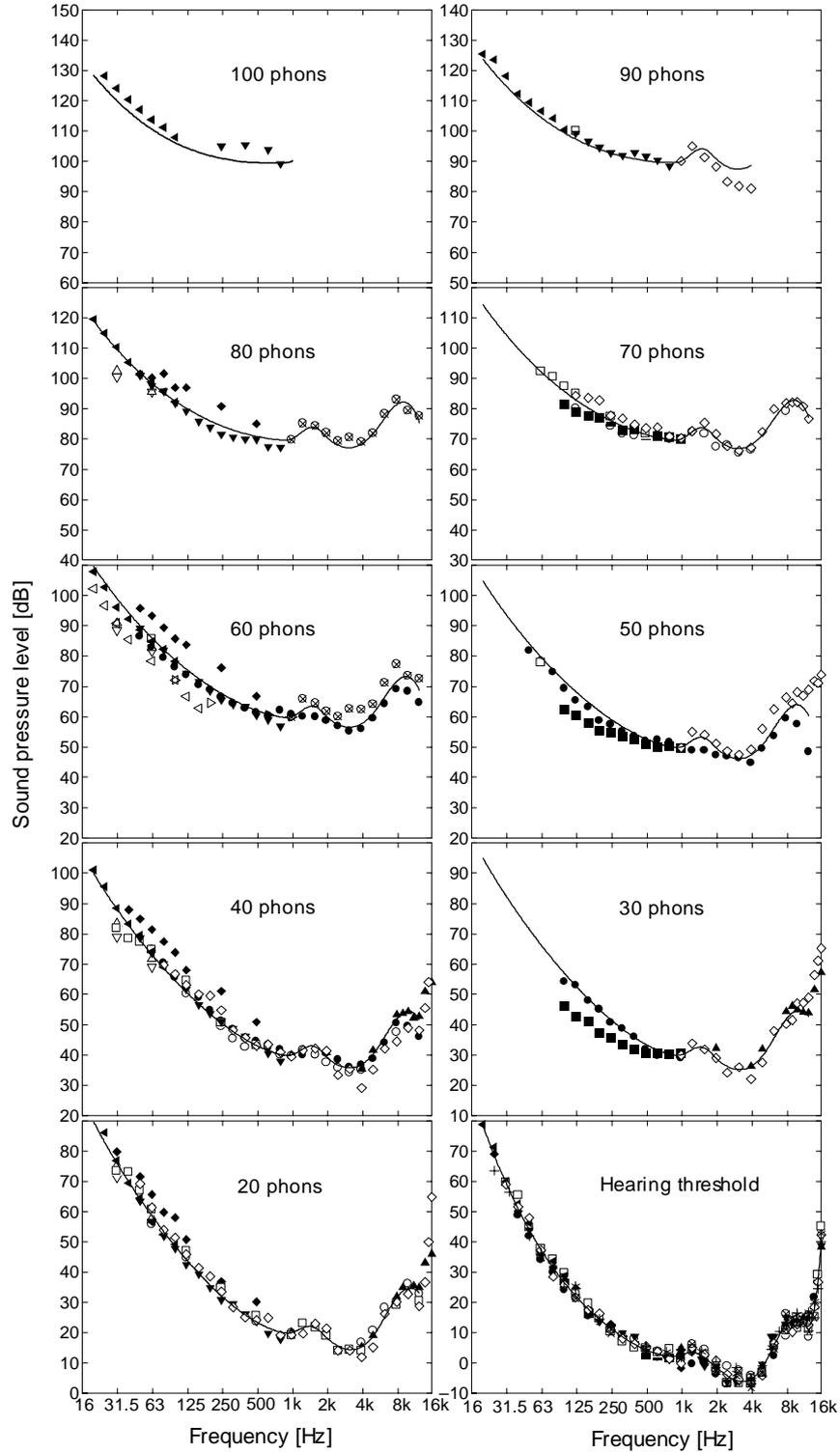
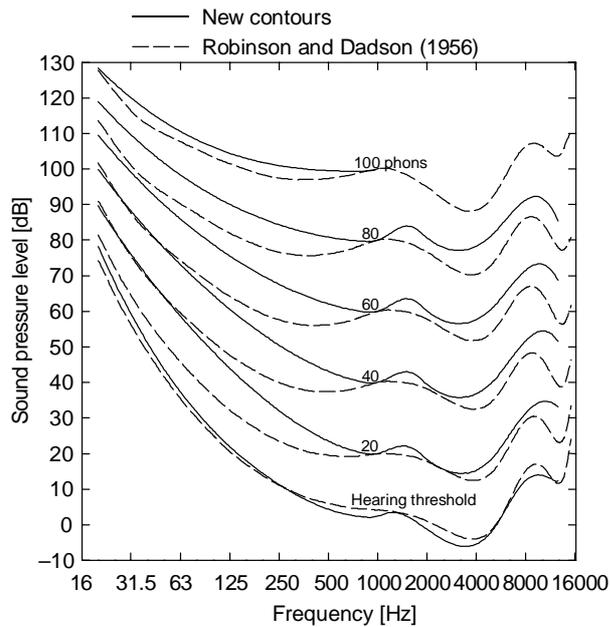


Fig. 2: Estimated new equal-loudness contours

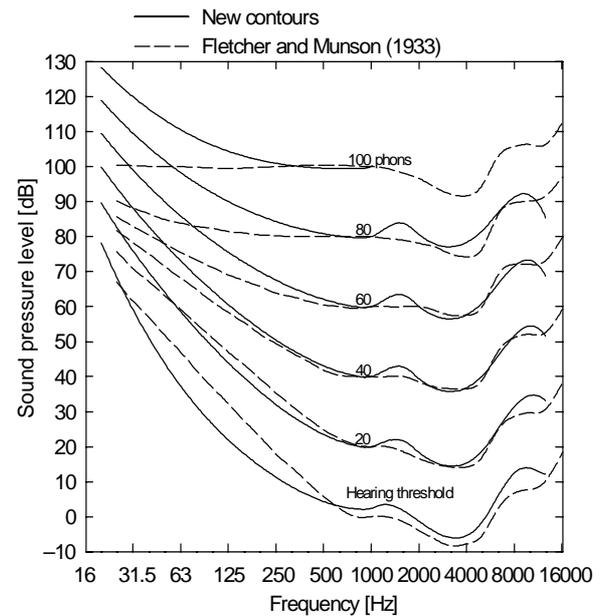
- ▽ Kirk (1983) PF
- △ Moeller and Andresen (1984) PF
- Betke and Mellert (1989)
- Suzuki et al. (1989)
- Fastl et al. (1989)
- ◆ Watanabe and Moeller (1990)
- ▲ Poulsen and Thøgersen (1994)
- ▼ Lydolf and Moeller (1997)
- ◀ Lydolf and Moeller (1997) PF
- Takeshima et al. (1997)
- ▷ Bellmann et al. (1999)
- ◁ Bellmann et al. (1999) PF
- ◇ Takeshima et al. (2001)
- ⊗ Takeshima et al. (2002)
- + Robinson and Dadson (1956) HT
- ∧ Teranishi (1965) HT
- × Brinkmann (1973) HT
- \* Vorlander (1991) HT
- \* Poulsen and Han (2000) HT
- Estimated contour



**Fig. 3:** Estimated new equal-loudness contours and the basic data for the contours



**Fig. 4:** Estimated new contours and the contour proposed by Robinson and Dadson[1] (former standard)



**Fig. 5:** Estimated new contours and the contour proposed by Fletcher and Munson [3]

hearing threshold curve, the 10-phon contour is also drawn by a dotted line.

Figure 3 compares directly the estimated contours with the basic data used for the derivation of the contours. Overall, the new equal-loudness contours provide a reasonable description of the experimental results.

#### 4. Features of Proposed Equal-loudness Contours

Figure 4 compares the newly estimated contours with those published by Robinson and Dadson [1] which was standardized in ISO226. Except for the threshold curve, the estimated equal-loudness contours lie distinctly above the contours by Robinson and Dadson [1]. The deviation between the two sets of contours is especially evident in the frequency region below 1 kHz over the loudness-level range from 20 to 80 phons.

Figure 5 compares the newly estimated contours with those published by Fletcher and Munson [3]. At 20 and 40 phons the contours of Fletcher and Munson [3] are similar to those estimated in the present study. However, at loudness levels above 40 phons their contours lie above the estimated contours at frequencies below 1 kHz. Despite some differences, it is important to note that the two sets of 40 phons contours in Fig. 5 closely agree across a wide range of frequencies. This contour, derived from Fletcher and Munson's [3] pioneering work, is used as the basis of the A-weighting function.

In the frequency region between 1 and 2 kHz a small peak amounting to a few decibels is seen in the estimated contours but it does not appear in the classic contours. (See Fig. 4 and 5.) A peak between 1 and 2 kHz has been consistently observed in recent work [11,15,17,19,20]. This peak seems to correspond to a small dip in the HRTF near this frequency range [30]. Quite possibly, a peak between 1 and 2 kHz is not evident in the *classic* studies because Fletcher and Munson [3] did not measure any

equal-loudness levels between 1 and 2 kHz whereas Robinson and Dadson [1] measured equal-loudness levels at only one point within this frequency region. As a result, this peak must be overlooked.

## 5. Summary

A new family of equal-loudness contours was estimated from 12 *recent* studies. An equation was derived to express the equal-loudness relation between pure tones at different frequencies. This procedure made it possible to draw smooth contours from discrete sets of data values. Except in the vicinity of the threshold and at very high SPLs, the equation provides a good description of the experimental results. In general, the estimated contours exhibit clear differences from those reported by Robinson and Dadson [1] which was the former standard contours in ISO226. The differences are most pronounced below 1 kHz. The classic contours by Fletcher and Munson [3] exhibit some overall similarity to the proposed estimated contours, but they also deviate from the proposed contours above 40 phons. Based on this research result, we made some draft standards of the new equal-loudness contours and proposed to ISO/TC 43 (Acoustics). The new standard finally was established on August 2003.

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